

Universalities of Triplet Pairing in Neutron Matter

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Abstract

The fundamental structure of the full set of solutions of the BCS 3P_2 pairing problem in neutron matter is established. The relations between different spin-angle components in these solutions are shown to be practically independent of density, temperature, and the specific form of the pairing interaction. The spectrum of pairing energies is found to be highly degenerate.

Since the discovery of superfluidity in liquid ^3He [1], great advances have been made in understanding the properties of superfluid systems with triplet pairing. In addition to the well-studied case of liquid ^3He below 2.6 mK [2–6], triplet pairing is expected to occur in neutron matter in the quantum fluid interior of a neutron star [7–11]. Neutrino cooling processes are strongly affected by $^3\text{P}_2$ pairing in this region [12], as is the vortex structure of the star and the coupling between core and crust [13,14]. A number of common features of superfluid, thermodynamic, and magnetic properties of different pair-condensed systems have been revealed by analyses based on symmetry principles [4–6], and further analytical insights have been gained near the critical temperature T_c by application of the Ginzburg-Landau approach [3]. However, new universalities of triplet pairing may be uncovered by a direct attack on the BCS gap equation, as we shall now demonstrate.

The purpose of this letter is to identify fundamental solutions of the triplet pairing problem in neutron matter and elucidate their structure and their relationships. If two identical spin- $\frac{1}{2}$ fermions are paired with a nonzero total momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$, the ordinary S -wave gap equation is converted into a system of coupled integral equations. In the standard notation [8,9], we have the expansion $\Delta(\mathbf{p}) = \sum \Delta_{LJ}^M(p) G_{LJ}^M(\mathbf{n})$ of the gap in the spin-angle matrices $G_{LJ}^M(\mathbf{n}; s_1, s_2) = \sum C_{\frac{1}{2}\frac{1}{2}s_1s_2}^{1Ms} C_{1LM_sML}^{JM} Y_{LM_L}(\mathbf{n})$, with $(\Delta_{LJ}^M(p))^* = (-1)^{J-M} \Delta_{LJ}^{-M}(p)$ assuming time-reversal invariance. The particle-particle interaction has the corresponding expansion $V(\mathbf{p}, \mathbf{p}') = \sum \langle p | V_{LJ}^{L'J} | p' \rangle G_{LJ}^M(\mathbf{n}) G_{L'J}^{M*}(\mathbf{n}')$, with $|L - L'| \leq 2$ in the case of tensor forces. The generalized BCS system then reads [9]

$$\Delta_{LJ}^M(p) = \sum_{L'L_1J_1M_1} (-1)^\Lambda \int \langle p | V_{LJ}^{L'J} | p' \rangle S_{L'JL_1J_1}^{MM_1}(\mathbf{n}') \frac{\Delta_{L_1J_1}^{M_1}(p')}{2E(\mathbf{p}')} \tanh \frac{E(\mathbf{p}')}{2T} d\tau', \quad (1)$$

where $\Lambda = L - L_1 + 1$, $d\tau = p^2 dp d\mathbf{n} \equiv d\tau_0 d\mathbf{n}$, and $S_{L'JL_1J_1}^{MM_1}(\mathbf{n}) = \text{Tr} [G_{LJ}^{M*}(\mathbf{n}) G_{L_1J_1}^{M_1}(\mathbf{n})]$ accounts for the summation over spin variables. The energy denominator $E(\mathbf{p}) = [\xi^2(p) + D^2(\mathbf{p})]^{1/2}$ involves the single-particle excitation energy $\xi(p)$ of the normal system and a gap function whose square is constructed as

$$D^2(\mathbf{p}) = \frac{1}{2} \sum_{LJM L_1J_1M_1} \Delta_{LJ}^{M*}(p) \Delta_{L_1J_1}^{M_1}(p) S_{LJL_1J_1}^{MM_1}(\mathbf{n}). \quad (2)$$

The pairing parameter Δ measuring the gap value at the Fermi surface is given by $\Delta^2 = \int D^2(p_F \mathbf{n}) d\mathbf{n} / 4\pi$.

Due to the nonlinearity of the gap equation (1), one must in general deal with off-diagonal effects in both the total (J, J_1) and the orbital (L, L', L_1) angular momentum quantum numbers. However, for the present we follow the usual practice dating back to Ref. [2] and suppress these effects, thus allowing for superposition of spin-angle components only in the magnetic quantum number M . The analysis is greatly facilitated by a generalization of the separation method developed for S-wave pairing in Ref. [15]. Thus, defining $\phi_{LJ}(p) = \langle p | V_{LJ}^{LJ} | p_F \rangle / v_F$ and $v_F = \langle p_F | V_{LJ}^{LJ} | p_F \rangle$, we employ the decomposition

$$\langle p | V_{LJ}^{LJ} | p' \rangle = v_F \phi_{LJ}(p) \phi_{LJ}(p') + W_{LJ}(p, p') \quad (3)$$

of the relevant pairing matrix into a separable portion and a remainder $W_{LJ}(p, p')$ that vanishes identically when either argument is on the Fermi surface. Integrals containing W_{LJ} as a factor are guaranteed to receive their overwhelming contributions some distance from the Fermi surface. In such integrals, the replacements $E(p, \mathbf{n}) \rightarrow |\xi(p)|$ and $\tanh(E/2T) \rightarrow 1$ are justified to high accuracy, errors in neutron matter being of relative order $D^2(\mathbf{p})/\epsilon_F^2 \sim 10^{-6}$, where ϵ_F is the Fermi energy.

Substituting (3) into (1) and invoking the orthogonality relation $\int S_{LJLJ}^{MM_1}(\mathbf{n}) d\mathbf{n} = \delta_{MM_1}$, the gap equations are recast as

$$\Delta_{LJ}^M(p) + \int \frac{W_{LJ}(p, p')}{2|\xi(p')|} \Delta_{LJ}^M(p') d\tau'_0 = v_F B_{LJ}^M \phi_{LJ}(p), \quad (4)$$

$$B_{LJ}^M = - \sum_{M_1} \int \phi_{LJ}(p) S_{LJLJ}^{MM_1}(\mathbf{n}) \frac{\Delta_{LJ}^{M_1}(p)}{2E(\mathbf{p})} \tanh \frac{E(\mathbf{p})}{2T} d\tau. \quad (5)$$

The quantities B_{LJ}^M are merely numerical factors. Consequently, the p dependence of all gap components is seen to be identical. Specifically, we may write $\Delta_{LJ}^M(p) = D_{LJ}^M \chi_{LJ}(p)$, where the shape factor $\chi_{LJ}(p)$ obeys an integral equation

$$\chi_{LJ}(p) + \int W_{LJ}(p, p') \frac{\chi_{LJ}(p')}{2|\xi(p')|} d\tau'_0 = \phi_{LJ}(p) \quad (6)$$

of the same form as in the singlet case [15]. To determine the amplitude D_{LJ}^M , we note that $\chi_{LJ}(p_F) = \phi_{LJ}(p_F) = 1$ since $W_{LJ}(p_F, p') = 0$. Therefore $\Delta_{LJ}^M(p_F) = D_{LJ}^M$, and Eq. (4) implies $D_{LJ}^M = v_F B_{LJ}^M$ while Eq. (5) gives

$$D_{LJ}^M = -v_F \sum_{M_1} \int \phi_{LJ}(p) S_{LJLJ}^{MM_1}(\mathbf{n}) D_{LJ}^{M_1} \frac{\chi_{LJ}(p)}{2E(\mathbf{p})} \tanh \frac{E(\mathbf{p})}{2T} d\tau. \quad (7)$$

The system (6)–(7) is more convenient for solution than the original equations (1), since the problem has been divided into (i) evaluation of the M -independent shape factor $\chi_{LJ}(p)$ from the nonsingular linear integral equation (6), and (ii) determination of the structure coefficients D_{LJ}^M from the nonlinear equation (7), where the log-singularity has been isolated.

Henceforth we specialize to the case $L = S = 1$, $J = 2$, this being the most favored uncoupled channel for pairing in neutron matter at densities prevailing in the quantum fluid interior of a neutron star ($k_F = p_F/\hbar \sim 2 \text{ fm}^{-1}$), where the $^1\text{S}_0$ gap has already closed [8,15]. The arguments are simplified if we adopt $D_{12}^0 \equiv \delta$ as a scale factor, write $D_{12}^{M \neq 0} \equiv (\lambda_M + i\kappa_M)\delta/\sqrt{6}$, and introduce a “structure function”

$$\begin{aligned} d^2(\mathbf{n}) = & 16\pi D^2(\mathbf{p})/\chi_{12}^2(p) = \delta^2[(1 + \lambda_2)^2 + \kappa_1^2 + \kappa_2^2 + (\lambda_1^2 - 4\lambda_2 - \kappa_1^2)x^2 \\ & - 2(\lambda_1 + \lambda_1\lambda_2 + \kappa_1\kappa_2)xz + (3 + \lambda_1^2 - \lambda_2^2 - 2\lambda_2)z^2 \\ & + 2(2\kappa_2 - \kappa_1\lambda_1)xy + 2(\kappa_1 + \lambda_1\kappa_2 - \lambda_2\kappa_1)yz], \end{aligned} \quad (8)$$

where $x = \sin\theta \cos\varphi$, $y = \sin\theta \sin\varphi$, and $z = \cos\theta$. After separation of the real and imaginary parts of the $D_{12}^{M \neq 0}$ in Eq. (7), we arrive at the set of equations

$$\begin{aligned} \lambda_2 &= -v_F[\lambda_2(J_0 + J_5) - \lambda_1 J_1 - \kappa_1 J_2 - J_3], \\ \kappa_2 &= -v_F[\kappa_2(J_0 + J_5) - \kappa_1 J_1 + \lambda_1 J_2 + J_4], \\ \lambda_1 &= -v_F[\lambda_1 J_6 - (\lambda_2 + 1)J_1 + \kappa_2 J_2 - \kappa_1 J_4/2], \\ \kappa_1 &= -v_F[\kappa_1 J_7 - \kappa_2 J_1 - (\lambda_2 - 1)J_2 - \lambda_1 J_4/2], \\ 1 &= -v_F[-(\lambda_1 J_1 - \kappa_1 J_2 + \lambda_2 J_3 - \kappa_2 J_4)/3 + J_5], \end{aligned} \quad (9)$$

which, in angular content, is consistent with the corresponding set in Ref. [9]. The integrals J_k are given by $J_6 = (J_0 + 4J_5 + 2J_3)/4$, $J_7 = (J_0 + 4J_5 - 2J_3)/4$, and, for $k = 1, \dots, 5$, by

$$J_k = \int f_k(\theta, \varphi) \frac{\phi_{12}(p)\chi_{12}(p)}{2E(\mathbf{p})} \tanh \frac{E(\mathbf{p})}{2T} d\tau, \quad (10)$$

with $f_0 = 1 - 3z^2$, $f_1 = 3xz/2$, $f_2 = 3yz/2$, $f_3 = 3(2x^2 + z^2 - 1)/2$, $f_4 = 3xy$, and $f_5 = (1 + 3z^2)/2$.

The system (9) has three one-component solutions [8,9] with $|M| = 0, 1$, and 2 . We preface our analytic exploration of multicomponent solutions with the following observation. Substitution of $\partial d^2(\theta, \varphi)/\partial\varphi$ for f_k in definition (10) must yield zero upon integration over φ . This identity implies a relation

$$\sum_{k=1}^4 c_k J_k = 0 \quad (11)$$

between the J_k integrals, with $c_1 = \kappa_1 + \lambda_1\kappa_2 - \lambda_2\kappa_1$, $c_2 = \lambda_1 + \lambda_1\lambda_2 + \kappa_1\kappa_2$, $c_3 = 2\kappa_2 - \kappa_1\lambda_1$, and $c_4 = 2\lambda_2 - (\lambda_1^2 - \kappa_1^2)/2$. Now observe that if the first equation of (9) is multiplied by $2\kappa_2$, the second by $2\lambda_2$, the third by κ_1 , and the fourth by $-\lambda_1$, and the results of the last three operations are subtracted from that of the first, relation (11) is reproduced. Thus only four of the five equations in (9) are truly independent and hence any one of the parameters λ_1 , λ_2 , κ_1 , κ_2 can be chosen arbitrarily. We take $\kappa_1 = 0$. With this choice, solutions of (9) are necessarily even functions of λ_1 , so attention may be focused on the sector $\lambda_1 \geq 0$.

The search for multicomponent solutions begins with the restricted case $\kappa_2 = 0$, for which $d^2(\mathbf{n})$ is independent of y . In Eq. (10), the integration of $f_k(\theta, \varphi)$ over y is then carried out using the formula $\sin\theta d\theta d\varphi = 2\delta(x^2 + y^2 + z^2 - 1)dx dy dz$. For $k = 1, 3$, this yields $2f_k(x, z)(1 - x^2 - z^2)^{-1/2}$ since f_1 and f_3 are independent of y , while both J_2 and $J_4 = 0$ vanish because f_2 and f_4 are odd in y and the y integral has symmetric limits. As a result, there remain only three independent equations,

$$\lambda_2 = -v_F[\lambda_2(J_0 + J_5) - \lambda_1 J_1 - J_3], \quad (12)$$

$$\lambda_1 = -v_F[\lambda_1(J_0/4 + J_5) - (\lambda_2 + 1)J_1 + \lambda_1 J_3/2], \quad (13)$$

$$1 = -v_F[-(\lambda_1 J_1 + \lambda_2 J_3)/3 + J_5]. \quad (14)$$

We first identify and verify a particular solution with $\lambda_1 = 0$, $\lambda_2 = 3$, for which the structure function (8) becomes $d^2(x, z) = 4\delta^2[4 - 3(x^2 + z^2)]$. The symmetry of this function with

respect to x and z implies the relation $3J_0 + 2J_3 = 0$, since the combination $3f_0 + 2f_3 = 6(x^2 - z^2)$ changes sign on interchange of x and z whereas d^2 and other factors within the integrand of (10) are left unchanged. Further, $J_1(\lambda_1 = 0, \lambda_2 = 3) = 0$ since $f_1 = xz$ is an odd function of x . Under these conditions, Eq. (13) is satisfied identically, while Eqs. (12) and (14) coincide and the resulting equation, $1 = -v_F(J_5 - J_3)$, determines δ . All other solutions of the set (9) are more degenerate. To illustrate this important feature, let us put $\lambda_2 = -1$. Then, at *any* λ_1 the structure function (8) is seen to take the factorized form $d^2(\mathbf{n}; \lambda_1, \lambda_2 = -1) = \delta^2(\lambda_1^2 + 4)(x^2 + z^2) = 24\pi\Delta^2(x^2 + z^2)$. The symmetry of d^2 in x and z again implies the relation $3J_0 + 2J_3 = 0$, while integration of $f_1 = xz$ over x gives 0 and therefore $J_1 = 0$. It follows that Eqs. (12)–(14) again coincide but now provide an equation $1 = -v_F[J_3/3 + J_5]$ that determines Δ^2 rather than λ_1 or δ individually. Here we have a striking example of the universal structure of solutions of the 3P_2 pairing problem, also manifested in the remaining solutions of the system (9).

These further solutions are found by implementing a rotation $R = (x = t \cos \alpha + u \sin \alpha, z = t \sin \alpha - u \cos \alpha)$. Expressing d^2 in terms of t and u and setting $\tan \alpha = \gamma$, one easily finds conditions

$$(\lambda_1^2 - 4\lambda_2)\gamma^2 + 2\lambda_1(1 + \lambda_2)\gamma + \lambda_1^2 - \lambda_2^2 - 2\lambda_2 + 3 = 0, \quad \lambda_1\gamma^2 - (\lambda_2 - 3)\gamma - \lambda_1 = 0 \quad (15)$$

under which d^2 becomes a function of t only. The choice $\gamma(\lambda_1, \lambda_2) = \gamma_0(\lambda_1, \lambda_2) = \lambda_1(1 + \lambda_2)/(4\lambda_2 - \lambda_1^2)$ meets both conditions provided

$$(\lambda_1^2 - 2\lambda_2 + 2)(\lambda_1^2 - 2\lambda_2^2 - 6\lambda_2) = 0. \quad (16)$$

Equation (16) embodies three branches of λ_2 versus λ_1 , which start as parabolas from $\lambda_1 = 0$ and $\lambda_2 = 1, 0$, and -3 . The structure function has t dependence $d^2(t) \propto 1 - t^2$ when the first factor of (16) vanishes and $d^2(t) \propto 1 + 3t^2$ when the second is zero. Calculating the integrals J_1 and J_3 by rotation of the x, z plane under R , we are led to the relations

$$\begin{aligned} (\lambda_1^2 - 4\lambda_2)J_1 + \lambda_1(\lambda_2 + 1)J_3 &= 0, \\ 3\lambda_1(1 + \lambda_2)J_0 - 2(\lambda_1^2 - 2\lambda_2^2 + 6)J_1 &= 0. \end{aligned} \quad (17)$$

The first relation (for example) is verified as follows, noting that the integrand on its l.h.s. is proportional to $[(\lambda_1^2 - 4\lambda_2)f_1 + \lambda_1(\lambda_2 + 1)f_3]/(1 - t^2 - u^2)^{1/2}$. Substituting $f_1(t, u)$ and $f_3(t, u)$ and integrating over u , which can be done freely for any shape of $d^2(t)$, we obtain a result that is proportional to $(\lambda_1^2 - 4\lambda_2)\gamma + \lambda_1(\lambda_2 + 1)$ and therefore vanishes when $\gamma_0(\lambda_1, \lambda_2)$ is substituted. What is remarkable is that Eqs. (12)–(14) coincide when relations (17) are inserted, and once again these equations determine only Δ^2 . Thus, construction of the rotation R “kills two birds with one stone”: the condition (16) required to transform d^2 into one-dimensional form also specifies another set of solutions of our system. Within the constraint $\kappa_1 = 0$, these solutions possess a line degeneracy as opposed to the point (or nondegenerate) character of the solution $(\lambda_1 = 0, \lambda_2 = 3)$. In addition to the free choice made for κ_1 , one of the coefficients λ_i can be assigned arbitrarily.

We now allow κ_2 to have a nonzero value, thus bringing in the second of Eqs. (9). At $\lambda_1 = 0$, this equation becomes identical with the first of the set, as is seen with the aid of Eq. (11). The particular solution $(\lambda_1 = 0, \lambda_2 = 3)$ is then replaced by one with $\lambda_1 = 0$ and $(\lambda_2^2 + \kappa_2^2)^{1/2} = 3$, but the pairing energy remains unaltered, the relevant quantities being independent of the phase of the coefficient $D_{12}^2(\lambda_1 = 0)$. To find the other multicomponent solutions in the general case with $\kappa_2 \neq 0$ and $\lambda_1 \neq 0$, we may extend our previous tactic and apply a rotation in *three*-dimensional space so as to eliminate four terms in expression (8) and cast d^2 into a one-dimensional form. The three Euler angles are thereby fixed, implying the single relation

$$\kappa_2^2 = (1 + \lambda_2)(\lambda_1^2/2 - \lambda_2 + 1) \quad (18)$$

between λ_2 , κ_2 , and λ_1 , which can be shown to satisfy all of Eqs. (9). This relation defines two branches $\kappa_2(\lambda_1, \lambda_2)$. Starting at the plane $\kappa_2 = 0$, one branch grows out of the solution $\lambda_2 = -1$ while the other grows out of the parabola $\lambda_1^2/2 = \lambda_2 - 1$ contained in Eq. (16). (Accordingly, $\lambda_2 = -1$ and this parabola cannot be counted as independent solutions.) The two surfaces defined by (18) complete the set of states of the 3P_2 problem.

The solutions we have identified divide into two groups, the states within a group being

essentially degenerate in energy. This behavior is consistent with the numerical calculations reviewed in Ref. [8]. The group with lowest energy, having structure function $d^2(t) \propto 1 + 3t^2$, contains only nodeless states and consists of (i) the particular state $(\lambda_1 = 0, \sqrt{\lambda_2^2 + \kappa_2^2} = 3)$ and (ii) the states belonging to the branches of Eq. (16) starting at the points $(\lambda_1, \lambda_2) = (0, 0)$ and $(0, -3)$. The upper group contains the remaining states, having $d^2(t) \propto 1 - t^2$ and one node. The splitting between the two groups can be calculated (for example) as the splitting between the states $\lambda_2 = -1$ and $(\lambda_1 = 0, \lambda_2 = 3)$, henceforth labeled u and l respectively. Evaluating (14) for both states, one finds the relation $J_3^{(u)}/3 + J_3^{(l)} + J_5^{(u)} - J_5^{(l)} = 0$ between integrals of the form (10). In explicating this relation, we exploit the fact that the dominant contributions to the integrals $J_3^{(u)}$, $J_3^{(l)}$, and $J_5^{(u)} - J_5^{(l)}$ come from the range of p values adjacent to the Fermi surface, where $\chi_{12}(p)$ and $\phi_{12}(p)$ are effectively unity. One readily arrives at the analytical result

$$\ln \frac{\Delta_u^2}{\Delta_l^2}(T = 0) = \frac{2\pi}{9\sqrt{3}} + \frac{2}{3} - \ln 3 \simeq -0.028 \quad (19)$$

for the splitting of upper and lower states, in close agreement with Ref. [8]. Similar results are also available at finite temperature T .

The conclusions that follow from these exercises are that if the mixing of different L, J channels is neglected, (i) the 3P_2 gap spectrum is nearly degenerate and (ii) its structure, in terms of energy splittings between the different states, is a universal function of T/T_c , independent of any other input parameters including the density. In particular, the concrete form of the particle-particle interaction V was not used anywhere, so the structure and relations we have established retain their validity even when fluctuation and polarization corrections to the bare V are taken into account.

Finally, we return to the issue of nondiagonal contributions to the system (1) of gap equations, which arise principally from the 3P_2 – 3F_2 coupling, and outline a perturbative evaluation [2] of their effects. The r.h.s. of each equation of the set (9) is now perturbed by a small “nondiagonal” contribution. In the presence of these additional terms, the degeneracies found above are removed. The pairing energy no longer depends on $d^2(\mathbf{n})$ alone, and the

parameters λ_i, κ_i are fully determined. Consider, for example, the alteration of the parabolic branch contained in the relation (16), which may be measured by a new variable $\zeta = \lambda_1^2 - 2\lambda_2 + 2$. Both ζ and the change $\eta = \delta(\zeta) - \delta(\zeta = 0)$ of the scale factor δ are expected to be small; therefore in performing Taylor expansions of the $J_k(\zeta, \eta, \lambda_1)$ we need only retain terms linear in ζ or η . The original set of three equations (12)–(14) is replaced by three new ones, each of which takes the schematic linear form $\zeta A(\lambda_1) + \eta B(\lambda_1) = P(\lambda_1)$ with different choices of the functions A , B , and P , all referred to $\zeta = \eta = 0$. The small quantities ζ and η may be obtained from any pair of the equations, as functions of λ_1 . Substituting these functions into the remaining equation, we arrive at a closed form that determines the value of λ_1 , which was hitherto arbitrary. Estimation and analysis of available numerical results [8,9] indicate that the nondiagonal corrections to the universal relations we have derived for triplet pairing in neutron matter are small, maximally of order several percent of the splitting given by Eq. (19).

In summary, straightforward arguments based on a new separation method [15] for treating BCS-type gap equations have revealed the structure and energetics of the full set of solutions of the pairing problem in the uncoupled 3P_2 channel. In contrast to the Ginzburg-Landau scheme employed by Mermin [3], the present approach is applicable at any temperature T . The analysis shows that the structure of the solutions is in fact universal — independent of the temperature, the density, and the specific parameters of the interparticle potential, which affect only an overall scale factor in the pairing energies. The line of analysis we have followed transcends the problem considered here. An obvious future objective is to characterize the solutions occurring in the problem of superfluid ^3He . The structure function $d^2(x, y, z)$ is again bilinear in its variables, but the states for $L = S = 1$ with $J = 0, 1, 2$ contribute on an equal footing and the number of equations in the system analogous to (9) rises from five to nine. The treatment introduced herein could also be relevant to the description of superdeformed bands in atomic nuclei, if triplet P-wave pairing is responsible for this phenomenon as suggested in Ref. [16].

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REFERENCES

- [1] D. D. Osheroff, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. **28**, 885 (1972).
- [2] P. W. Anderson and P. Morel, Phys. Rev. **123**, 1911 (1961).
- [3] N. D. Mermin, Phys. Rev. **A9**, 868 (1974).
- [4] Y. Hasegawa, T. Usagawa, and F. Iwamoto, Prog. Theor. Phys. **62**, 1458 (1979).
- [5] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3* (Taylor & Francis, London, 1990).
- [6] G. E. Volovik, in *Helium Three*, ed. W. P. Halperin and L. P. Pitaevskii (North Holland, Amsterdam, 1990).
- [7] D. Pines and M. A. Alpar, Nature **316**, 27 (1985).
- [8] T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. Suppl. **112**, 27 (1993).
- [9] L. Amundsen and E. Østgaard, Nucl. Phys. **A442**, 163 (1985).
- [10] M. Baldo, J. Gugnion, A. Lejeune, and U. Lombardo, Nucl. Phys. **A515**, 409 (1990).
- [11] Ø. Elgarøy, L. Engvik, M. Hjorth-Jensen, and E. Osnes, Nucl. Phys. **A607**, 425 (1996).
- [12] D. Page and J. H. Applegate, Ap. J. Lett. **394**, L17 (1992).
- [13] J. A. Sauls, D. L. Stein, and J. W. Serene, Phys. Rev. D **25**, 967 (1982).
- [14] F. K. Lamb, in *Frontiers of Stellar Evolution*, ed. D. L. Lambert (Astronomy Society of the Pacific, San Francisco, 1991), p. 299.
- [15] V. A. Khodel, V. V. Khodel, and J. W. Clark, Nucl. Phys. **A598**, 390 (1996).
- [16] V. I. Fal'ko and I. S. Shapiro, Sov. Phys. JETP **64**, 706 (1986).